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"Furthermore, no one, though he speak with the tongues
of angels could keep people from using *tertium non datur*"

DAVID HILBERT... 'ON THE INFINITE'

[pg. 145 BENACERAF & PUTNAM]
PHILOSOPHY OF MATHEMATICS]

THEORY OF INTERMEDIATE CARDINAL NUMBERS.

9 - 11 - 71

Consider the equivalence $\aleph_{p/q} \equiv \sim p \vee \sim p$ when $\aleph_0$ and $\aleph_p$ or $\aleph_{\sim p}$ coincide.

Negation is seen to emerge $> \aleph_0$; which is why the realm of intermediate cardinal number is beyond tertium non datur!

And beyond the $\aleph_0$ confluence of propositions and negations.

Cf. tertium non datur must hold internally for $\aleph_0$'s realm.

For reality is neither p nor $\sim p$, and this is said of $\aleph_0$ number of confluent propositions.

It can be said of $2^{\aleph_0}$, etc., number of confluent propositions.
(But then $\aleph_{p/q} \not\equiv \sim p \vee \sim p$).

As truth and falsity are the issue of tertium non datur, $2^{\aleph_0} = C$ is neither true nor false.

But to say $2^{\aleph_0} = C$ is the empty set is to render it true.
Just as to say it is not the empty set is to render it true also.
(For $\aleph_{p/q} \equiv \sim p \vee \sim p \supset x : x \neq x$).

But $\aleph_{p/q} \equiv \sim p \vee \sim p \Rightarrow \hat{x}: x \neq x$ cannot
render Cantor's theorem true.

i.e. $\aleph_{p/q} < 2^{\aleph_0}$, if proven, may obtain a differing existence
for the empty-set.

What number of $\hat{x}: x \neq x$? (cf. $\aleph_{x: x \neq x} = \aleph_{p/q} < \aleph_{p/q}$)

Unlike Cantor's empty-set $\aleph_{p/q}$ does not obtain "vacuity without
number".

$\aleph_{p/q}, \aleph_{p/q}, \dots$ non-Cantorian intermediate transfinite numbers.

If the empty-set is a subset of every set it is a subset of the
universal-set. If the universal-set is confined by tertium non datur,
then the negation of tertium non datur is the empty-set; therefore
the negation of tertium non datur is a subset of the universal-set.

How large is the empty-set obtained by negating tertium non datur?

If it is as large as the universal-set (if tertium non datur and
the universal-set coincide), then the empty-set is not a proper
subset of every set but rather a subset; hence the universal-
set is without tertium non datur.

Also, substitute $\aleph_0$ for the universal-set.

If 2^{λ_0} exists and the empty-set is a subset of the universal-set, then λ_0 is not the universal-set. Hence a universal-set as large as λ_0 is an empty-set. As this set has no empty-set as a subset, the empty-set is not a subset of every set. If the empty-set is allowed an empty-set as a proper subset, then at least two empty-sets exist.

If tertium non datur totally defines the universal-set and the empty-set is reflexive, then the empty-set is not a subset of the universal-set. If the negation of tertium non datur is a set, then the empty-set is one of its subsets.

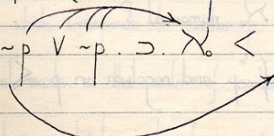
$p = \sim p$ or tertium non datur fails!

e.g. p 's entirety intersects $\sim p$'s particularity unless neither $p \vee \sim p$ is a boundary ($\therefore p = \sim p$).

If p 's entirety excludes (or negates) $\sim p$, tertium non datur fails.

If p 's entirety does not exclude (or negate) $\sim p$, tertium non datur fails (because "all $p = \sim p$ ").

Consider $\sim p \vee \sim p \rightarrow \lambda_0 < \lambda_{p/q}$



Negation in $p \leq \aleph_0$ and negation after $\aleph_0$.

i.e. not "p and negation in $p \leq \aleph_0$ ".

Tertium non datur is "an empty-set below $\aleph_1$ ".

$\aleph_{x:x \neq x}$: the negation of tertium non datur = $\aleph_0$.

Observe how $\aleph_{x:x \neq x}$ emerges for integers everywhere.

As you should observe how $\aleph_1$ emerges beyond $\aleph_{x:x \neq x} = \aleph_0$.

$\aleph_p = \aleph_{-p}$, because of negation after $\aleph_0$.

Theorem: $\aleph_{p/q} \equiv$ not "p and negation in $p \leq \aleph_0$ ".

So, minimally, there must be something "propositional" beyond the first infinity.

Is this "affirmation" beyond $\aleph_0$ numerical?

$\aleph_0$ number of negations of "p and negation in $p \leq \aleph_0$ ".

The empty-set is without tertium non datur.

$$2^{\hat{0}} - \hat{0} \neq 2^{\hat{0}-1}$$

State $2^{\hat{x}} = \lambda_{x:x \neq x}$

If $\hat{0} \subseteq A$, with A infinite, then 2^A carries an empty-set as exponent.

So, at some stage of multiplication, 2 grows into an empty-set.

By this reasoning $2^{\hat{x}} < \lambda_{x:x \neq x}$ "in emptiness".

i.e. if $\hat{x}: x \neq x = \overline{N}$ "in emptiness".

Not λ_0 empty-sets unless they be different.

12 - 11 - 71

The reality which corresponds to neither p nor $\sim p$ is not empty.

It is what exists as the realm of excluded tertium non datur.

Yet it has cardinal number (even as cardinal number exists for p and $\sim p$).

You must ask the question as to which transfinite number lies first beyond tertium non datur.

$$\aleph_0 < x \leq \aleph_{p/q}.$$

"Transfinite emptiness?" As a subset of what in which realm?

Without contradiction $\aleph_0$ cannot exist. [N.B.]

So, if $\aleph_{p/q}$ is contradictory, something from $\aleph_{p/q}$ goes to $\aleph_0$.

"If contradiction is merged into being, $\aleph_{p/q}$ exists".

If the non-contradictory excludes the contradictory (and $\aleph_0$ is consistent), then $\aleph_0$ and $\aleph_{p/q}$ (if $\aleph_{p/q}$ is contradictory) have only an empty-set in common.

$\aleph_0$ and $\aleph_{p/q}$ have negated tertium non datur in common.

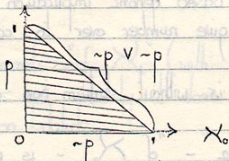
So tertium non datur, when negated, is infinite.

Negated tertium non datur is found in $\aleph_0$ as the empty-set.

Examine $\hat{x}: x \neq x \notin \aleph_0$, (only $\hat{x}: x \neq x \subset \aleph_0$).

$X_2 = 2^{(2^{X_0})}$, so where is tertium non datur?

$X_0 < 2^{X_0} < C_{X_0}$, likewise.



i.e. by which analogy the set of real numbers is without tertium non datur.

Consider Cantor's diagonal as $X_{p/q}$! (From the assumption that $X_{p/q}$ coincides with the hypotenuse).

As if the diagonal were from $X_{p/q}$ to X_0 (or the sets of X_0).

"Being as a model for tertium non datur".

Only then 2^{X_0} , $2^{(2^{X_0})}$, exclusions of anything but tertium non datur would exist.

(And then such an ascending sequence would not form tertium non datur).

We obtain λ_0 by generic implication (by holding to p and $\sim p$ within the confines of tertium non datur).

But have we not then based generic implication within λ_0 ? And to do this is to give number over to confinement.

Non-Cantorian set theory is without tertium non datur ($\geq \lambda_0$).

Whilst the "genitive existence" - "of λ_0 " - is possibly without tertium non datur.

Standard set theory is with tertium non datur and the axiom of selection.

Restricted set theory is with tertium non datur but the axiom of selection is neither true nor false.

Non-standard set theory, with tertium non datur, negates one form or other of the axiom of selection or the continuum hypothesis.

Ordinary set theory is restricted set theory without the necessity of constructible sets. (Constructible set theory is a model for ordinary set theory).

So ordinary set theory is without the axiom of selection.

Consider the genitive existence of $\lambda x: x \neq x$. If tertium non datur is "of λ " and, for all x , $x \neq x$ is "of λ " - i.e. that there are λ reflexive elements - then $\lambda x: x \neq x$ is self-contradictory.

$\lambda x: x \neq x$ has elements "of λ " (with tertium non datur) - i.e. that many - yet, plainly it does not frequent the realm of tertium non datur.

$\lambda x: x \neq x$ negates the axiom of selection as its elements appear from empty-sets.

If sets are distinct and the empty set is a subset of every set, $\lambda x: x \neq x$ is of necessity disjoint for its elements.

As the axiom of selection is for non-empty, disjoint sets, so $\lambda x: x \neq x$ is for empty, disjoint sets.

So restricted set theory must not contain the theorem, "the empty set is a subset of every set", as such a theorem is without tertium non datur.

How many empty-sets "of X_0 "?

For X_0 (by association of subsets) 2^{X_0} many, minimally.

As $X_0 \supset \cdot$ tertium non datur $2^{X_0} - X_0$ (sub-)sets, minimally, are without tertium non datur and 2^{X_0} maximally.

$$2^{X_0} - X_{x:x \neq x}.$$

If the axiom of selection is "of X_0 " non-empty, disjoint sets, how many empty-(subsets) exist?

Presumably they should be "proper subsets" (or else the axiom of selection is negated). Yet if they are proper subsets the axiom of selection is negated (as 2^{X_0} proper subsets is beyond the cardinality of the axiom of selection with tertium non datur).

If tertium non datur is self-contradictory, then so is the empty-set. (So restricted set theory and ordinary set theory become Non-Cartesian).

18-11-71

Number must not have inadequate existence. N.B.

Yet control by signs is inadequate when tertium non datur $\Leftarrow X_0$.

"Are numerical signs empty?" Only if all of notation is denumerably-infinite. (For to what could such notation refer but nothing else?)

Within the confines of tertium non datur, existence is inadequate.

2^{X_0} precedes X_0 , and 2^{X_0} is without tertium non datur.

$X_{x:x \neq x} \equiv \bar{N}$, i.e. that much vacuity "of X_0 ", unless the negation of tertium non datur is "of X_0 ".

Tertium non datur must negate X_0 -much "referential emptiness".

Notice how the axiom of selection is not "of X_0 ".

Similarly, how the continuum hypothesis is not "of X_0 ".

Yet the continuum hypothesis is expected to carry tertium non datur beyond X_0 .

Just as emptiness (in the model of a set) is carried beyond X_0 by mathematical induction, or by a "true" implication.

When mathematical induction is within the confines of tertium non datur - and, hence, of X_0 ; and when "truth" is of tertium non datur and not of $> X_0$.

So either truth is of $\leq X_0$ or not.

If it is, then truth is not of 2^{X_0} nor even of $X_1, X_2, \dots$

If it is not, then finite and denumerably-infinite mathematics is without truth.

To deny X_1 is to recreate X_0 as an empty-set (..... " X_0 follows").

19-11-71

Answer X_3 in arithmetics of infinity.

$$2^{(2^{X_0})} = X_3.$$

With the genuine existence for tertium non datur "of $\aleph_0$ ",
the axiom of selection is distinct from tertium non datur.

Should the axiom of selection and tertium non datur hold a set (or cardinal number) in common, such a set is empty or a negation "of $\aleph_0$ " (or it is a negation of tertium non datur).

Such a negation, a negation in common, is not "of $\aleph_0$ " internally; it is not denumerably infinite as it would comprise a set "of tertium non datur".

So, the axiom of selection is not a model "of $\aleph_0$ ".

Similarly, the negation of the continuum hypothesis is not "of $\aleph_0$ ". (Which says it is not constructible by "matching" sets).

"Then both the axiom of selection and the continuum hypothesis are situated in a realm of negation?"

Unless they are situated "between" $\aleph_0$ and $\aleph_{p/q}$ or $2^{\aleph_0}$.

"Then is the axiom of selection's set $\aleph_0 < \alpha < 2^{\aleph_0}$?"

Unlike $\aleph_0$ the selection set is not made of tertium non datur.

Neither is the selection set "of $\aleph_0$ ".

Is the selection set $\aleph_{1/4}$? (Not if it is "of $2^{\aleph_0}$ ").

21-11-71.

Only the empty-set in tertium non datur is identically negated (with p).

Nothing in tertium non datur is identically negated.

With the negation of tertium non datur identical negation (outside tertium non datur and inside tertium non datur) is empty.

Negation, therefore, outside of tertium non datur is necessarily $> \aleph_0$, i.e. $\aleph_0$ "of tertium non datur".

So there exists a cardinal number $> \aleph_0$.

So if $\aleph_0$ does not "exhaust" the selection set and if the selection set is without tertium non datur, then either $\aleph_0$ cannot be selected or tertium non datur is selected from the negation of tertium non datur.

"Is tertium non datur from the negation of tertium non datur?"

No, as the negation in tertium non datur is not the negation of tertium non datur.

Identity of negation or the negation of tertium non datur (but not both).

Negation of $\aleph_0$ is outside tertium non datur.

So, if tertium non datur is negated then either $\aleph_0$ is without tertium non datur or there is no (Cantorian) least transfinite cardinal number.

If $\aleph_0$ exists, contradiction exists (of something by p or $\sim p$), and if contradiction exists, the contradiction "of $\aleph_0$ " is $\geq \aleph_0$.

Contradiction $> \aleph_0$ is the opposite of tertium non datur.

$\aleph_0/p/q$ is non-constructible (and hence not denumerably-infinite) "of $\aleph_0$ ". (For if $\aleph_0/p/q$ were denumerably-infinite $\aleph_0$ would exist per se).

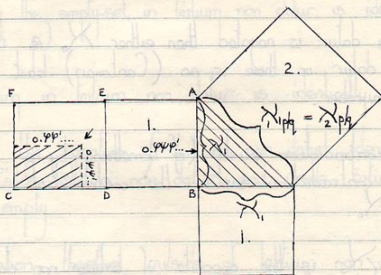
If p and $\sim p$ are identical in size $\leq \aleph_0$, $\aleph_1$ creates a second of contradiction.

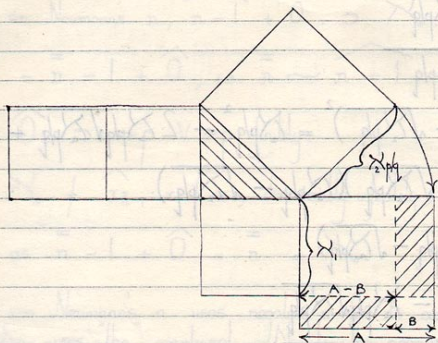
Negate X_0 's "consistency" beyond X_0 .

Negation outside tertium non datur, if affirmed "of $X_1 p/q, X_1$,
 $X_2 p/q \dots X_n p/q$ ", asserts the confluence of p and $\sim p$.

Which is to say, neither tertium non datur nor the law of contradiction hold beyond X_0 .

22-11-71





$$(A - B)^2 = A^2 - 2AB + B^2$$

$$\Rightarrow (x_2 p/q - x_1)^2 = x_2 p/q^2 - 2x_2 p/q x_1 + x_1^2$$

$$(x_2 p/q - x_1)(x_2 p/q - x_1)$$

$$1) x_2 p/q (x_2 p/q - x_1) = x_2 p/q^2 - x_2 p/q x_1$$

$$2) -x_1 (x_2 p/q - x_1) = -x_2 p/q x_1 + x_1^2$$

$$\therefore x_2 p/q^2 - 2x_2 p/q x_1 + x_1^2 = x_2 p/q^2 - 2x_2 p/q x_1 + x_1^2$$

$$X_1 = \sqrt{X_2 p/q}$$

$$X_1^2 \neq \sqrt{X_2 p/q}$$

$$\text{i.e. } (X_2 p/q - \sqrt{X_2 p/q})^2 = X_2 p/q^2 - 2 X_2 p/q \sqrt{X_2 p/q} + \sqrt{X_2 p/q}$$

$$(X_2 p/q - \sqrt{X_2 p/q})(X_2 p/q - \sqrt{X_2 p/q})$$

$$1) X_2 p/q (X_2 p/q - \sqrt{X_2 p/q})$$

$$= X_2 p/q^2 - X_2 p/q \sqrt{X_2 p/q}$$

$$2) -\sqrt{X_2 p/q} (X_2 p/q - \sqrt{X_2 p/q})$$

$$= -X_2 p/q \sqrt{X_2 p/q} + X_2 p/q$$

$$X_2 p/q^2 - (X_2 p/q \sqrt{X_2 p/q} - X_2 p/q \sqrt{X_2 p/q}) + X_2 p/q \quad \text{invalid,}$$

$$\therefore X_2 p/q^2 - X_2 p/q \sqrt{X_2 p/q} - X_2 p/q \sqrt{X_2 p/q} + X_2 p/q$$

$$= X_2 p/q^2 - 2 X_2 p/q \sqrt{X_2 p/q} + X_2 p/q$$

$$\therefore X_1^2 \neq \sqrt{X_2 p/q} \quad (\therefore X_1 \neq X_1^2), \text{ yet } X_1 = \sqrt{X_2 p/q}$$

J. von Neumann's n , $\widehat{n-1} + \widehat{0} \supset X_0$.

$X_0 = \widehat{n} - 1 + \widehat{0}$, i.e. $\widehat{n} \sim \widehat{n-1} + \widehat{0} \supset X_0$.

But $\widehat{0} \supset \sim p \vee \sim p$ finally.

Cf. $X_0 \neq \widehat{x_{\widehat{n}-1}}: x \neq x$, denumerably-infinitely,

i.e. as $\widehat{n-1} + \widehat{0} = \widehat{n} = X_0$.

Now von Neumann's n uses negative n for its own generation; it also uses "the" empty-set. Negate the negative n and proliferate "the" empty-set,

i.e. $x \neq x \supset 0$, $\widehat{x_1}: x \neq x \supset 1$, and so forth: (instead of n issuing from preceding n , let the first empty-set generate zero, the second empty-set generate one, and so forth).

Then $\widehat{x_{x_0}}: x \neq x \supset \widehat{n}$,

(so, $X_0 = \widehat{x_{\widehat{n}}}: x \neq x$ is not at issue).

Then $\widehat{x_{x_0+1}}: x \neq x = X_0$. ($X_0 + 1 = X_0$).

$$X_0 + \hat{x}_1: x \neq x = X_0.$$

Therefore X_0 's "following" issue, $\hat{x}_n: x \neq x$ (the natural numbers) is without *tertium non datur*.

Neither is X_0 circular (unlike von Neumann's n which is),
as $x_{x_0+1}: x \neq x = X_0$.

25-11-71.

$$\hat{x}_{x_0+1}: x \neq x \quad (\text{Axiom des Unendlichen}).$$

$\hat{x}_{x_0+1}: x \neq x$, considered as the "generation of emptiness" reversed,

i.e. $0, \hat{0}, \hat{\hat{0}} \dots = n$, reversed.

Therefore the number of sets of empty-sets (including zero) is not primitive, and neither is the concept of natural number.

cf. $\sum_{k=0}^{\infty} -1^k$ as a generative plus and minus concept.

"Zusammenfassung" of emptiness for sets is without *tertium non datur*.

Consider $\hat{x}_{x_0+1}: x \neq x = X_0$ when $\hat{x}_{x_0+1}$ is an empty-set.

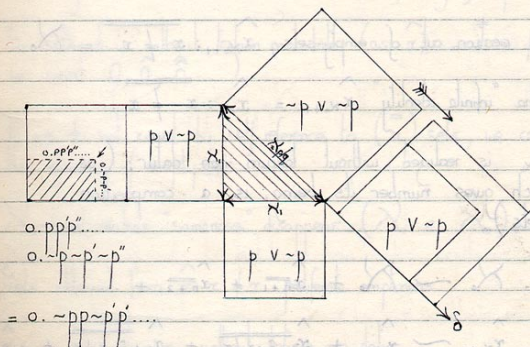
$\aleph_0$ is realized by appeal to empty-sethood.

Hence $\aleph_0$ is realized by appeal to the negation of tertium non datur.

"First infinity to give transfinite number, then tertium non datur".

Cantor's diagonal δ passes beyond $\aleph_2$ p/q?

Cantor's diagonal δ passes beyond $\sqrt{\aleph_2}$ p/q! i.e.:



$$\text{Cf. } \sum_{k=0}^{\infty} -1^k p v \sim p > \aleph_0$$

($\aleph_1$ "logical space" of two-dimensions).

$$\therefore X_0 + \hat{x}_2 : x \neq x = X_0 < \sim p \vee \sim p, \text{ assumed};$$

$$X_{p/q} = \sim p \vee \sim p < X_1 (p \vee \sim p), \text{ assumed};$$

$$\sqrt{X_2}_{p/q} (p \vee \sim p) < X_2_{p/q} = \sim p \vee \sim p.$$

$\therefore$ If X_1 possesses a form of tertium non datur, more than one "middle-third" exists.

26-11-71

X_0 is a cection out of empty-sets, $\hat{x}_{X_0+1} : x \neq x = X_0$.

We have the "infinite identity" $\hat{x}_{X_0+1} = \hat{x}_{X_0} : x \neq x$.

$\hat{x}_{X_0+1} = \hat{x}_{X_0}$ is realized without tertium non datur, (as x_{X_0+1} , which gives number its being, is a composition of empty-sets).

$$\hat{x}_{X_0+1} = X_0 \sim X_0 + \hat{x}_{\overline{n+1}} + \hat{x}_{\overline{n+2}} + \dots$$

$$\text{i.e. } \hat{x}_{X_0+1} = \hat{x}_{X_0+1} \sim x_{X_0+1} + \hat{x}_{\overline{n+\hat{x}_2 : x \neq x}} + \hat{x}_{\overline{n+\hat{x}_1 : x \neq x}} + \dots$$

X_0 number of empty-sets (from $\hat{x}_{X_0+1} : x \neq x$),

$$X_1 \cup \hat{x}_{x_0+1} : x \neq x$$

Notice integer form beyond X_0 is empty.

$$\hat{x}_{(x_0-1)} \cup \hat{0} = \hat{x}_{(x_0-1)} \cup \overline{x \neq x}$$

$$X_0 + n = X_0, \text{ because } n \text{ is empty.}$$

$$X_0 - n = X_0, \text{ likewise}$$

Evidently X_0 is numberless (i.e. "Zusammenfassung" of emptiness).

So sethood is without tertium non datur (as $\hat{x}_n = \overline{n} : n \neq n$ as we count $0, \hat{0}, \hat{\hat{0}} \dots$).

If we do not count by self-reference to $(\overline{n})$ sets, we count by the reference of empty-sets.

X_0 -hood makes "sameness" disappear. ($X_0 \in X_0$).

Sets obtained from $\hat{x} : x \neq x$ proliferate emptiness

$\hat{x}_{x_0+1} : x \neq x$ is without tertium non datur

"All else but $\hat{x}_{x_0+1} : x \neq x$, with equivalences, is from numbered source

So number, to escape circularity, must issue from $\hat{x}_{x_0}$, etc.

e.g. "Zusammenfassung" of emptiness in $2 \mathcal{X}_{2p/q} \sqrt{\mathcal{X}_{2p/q}}$;

$\hat{x}_3: x \neq x$ of $2 \mathcal{X}_{2p/q} \sqrt{\mathcal{X}_{2p/q}}$ and not only for the characteristic digit.

As $\mathcal{X}_{2p/q} \sqrt{\mathcal{X}_{2p/q}}$ is multinomial it does not come about by simple digitalisation.

$$\hat{x}_3 = x \neq x, \widehat{x \neq x}, \widehat{\widehat{x \neq x}},$$

i.e. "two of" $\mathcal{X}_{2p/q} \sqrt{\mathcal{X}_{2p/q}}$.

So we have to see "one of" it in $\mathcal{X}_{2p/q}(\mathcal{X}_{2p/q} - \mathcal{X}_1)$;
and "one of" it in $-\mathcal{X}_1(\mathcal{X}_{2p/q} - \mathcal{X}_1)$.

Amidst digitalisation we must apprehend $\sim p \vee \sim p$, whilst there is no call to apprehend tertium non datur first.

Contradiction is empty for $\hat{x}_{x_0+1}: x \neq x$, but not necessarily an emptyset.

It is not evident that $\hat{x}_{x_0+1}: x \neq x$ issues from $\mathcal{X}_1$.

